FULL NONLINEAR ANALYSIS OF DETECTOR CIRCUITS USING RITZ-GALERKIN THEORY

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ABSTRACT

The Ritz-Galerkin method provides an algebraic approach to the large-signal analysis of exponential-diode detector circuits that avoids the limitations imposed by truncated series approximations. The resulting closed-form algebraic expressions account for the details of operation in both the square-law and linear regions, and are consistent with numerical simulations. At high signal levels the results agree with the predictions of a piecewise-linear analysis.

INTRODUCTION

The current success of numerical simulation techniques obscures the ability of approximate algebraic methods to provide *global* insights into the behavior of certain nonlinear circuits. It is often cumbersome to obtain such insights numerically because of the numerous simulations required.

Here the Ritz-Galerkin (RG) method [1]-[5] is used to obtain a closed-form algebraic solution that relates the video output voltage of a microwave detector circuit to the incident rf power. This solution provides a conceptual bridge between a conventional small-signal quasi-linear analysis of the "square-law" behavior [6], and a strictly high-level piecewise analysis of the "linear" behavior. It will be shown that the solution is valid over the full dynamic range from "square law" to "linear", and that the slope of the transfer function can exceed "square law" under certain conditions.

RITZ-GALERKIN ANALYSIS OF A DIODE DETECTOR

The diode in Fig. 1 is modeled as an exponential device obeying

$$i = I_s(e^{\Lambda v} - 1) \tag{1}$$

in series with a resistance R_s. In (1), $\Lambda = e/(nkT)$, n is the diode ideality factor, and k is Boltzmann's constant. The junction capacitance C_j is represented as voltage-invariant since it plays no important part in detector operation.

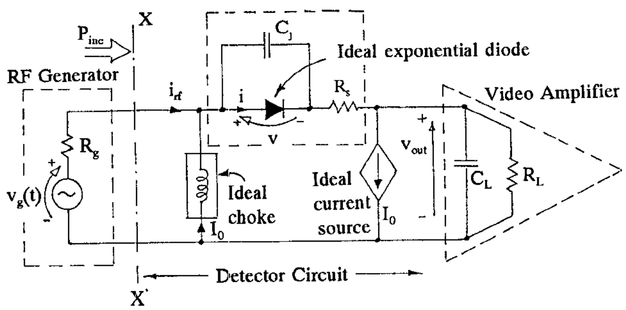

Figure 1. Simplified detector equivalent circuit for large-signal analysis by Ritz-Galerkin method. The diode is modeled as an ideal exponential diode with a series resistance R_s and junction capacitance C_j.

A relation is sought between the incident rf power P_{inc} and the video output voltage v_{out}. P_{inc} is the power impinging on the input port (X-X') of the detector circuit, regardless of its matching or nonlinearity. P_{abs} is that portion of P_{inc} actually absorbed by the nonlinear detector circuit. The rest of P_{inc} is reflected, and can include harmonics and possibly subharmonics of the input frequency.

Warner's small-signal detector analysis [6] assumed classical matching for maximum power transfer (MPT) to the diode, so that $P_{inc} = P_{abs}$. An MPT theorem for nonlinear resistive (memoryless) circuits has been obtained by Wyatt [7], but his generalization [8] to nonlinear circuits with memory leads to the requirement for the adjoint of a causal operator, which is anticipative and therefore nonphysical.

Practical detectors are normally operated in a non-MPT mode, and the question of matching for MPT does not arise. Accordingly, in the present analysis, the differential equation of the detector circuit is solved approximately, yielding a relation between the rf generator voltage $v_g(t)$ and the video voltage v_{out}. Then P_{inc} is found from $v_g(t)$.

The differential equation for Fig. 1 is

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$$a[\exp\{(x-y)-b(\mathring{y}+y)-k\zeta\}-1] \;+\; g[(\mathring{x}-\mathring{y})-b(\mathring{\mathring{y}}+\mathring{y})]=a\zeta+\mathring{y}+y \tag{2}$$

where $x = \Lambda v_g$ is the forcing function, and $y = \Lambda v_{out}$ is the video output. Other quantities are: $a = \Lambda R_L I_s$, $b = (R_g + R_s)/R_L$, $g = C_j/C_L$, $k = \Lambda R_s I_s$. $\zeta = I_0/I_s$ is a bias-current parameter. The symbols"o" and "oo" indicate $d/d\tau$ and $d^2/d\tau^2$ respectively, where $\tau = t/(R_L C_L)$. For an input at frequency ω,

$$x(\tau) = X cos(\nu\tau) \tag{3}$$

where $\nu = \omega R_L C_L$ and $X = \Lambda V_g$.

To apply the RG method, the differential equation is represented in the form

$$\xi[\; d/d\tau,\; y,\; x \;] \equiv 0 \tag{4}$$

where ξ is a nonlinear operator. The exact solution $y(\tau)$ is approximated by

$$\tilde{y}(\tau) = \sum_{k=1}^{N} a_k \psi_k(\tau) \tag{5}$$

where the $\psi_k(\tau)$ are linearly independent functions and the a_k are adjustable coefficients. In general

$$\xi[\; d/d\tau,\; \tilde{y},\; x \;] \;=\; \epsilon(\tau) \neq 0. \tag{6}$$

It can be shown [10] that the magnitude of the residual $\epsilon(\tau)$ is minimized by satisfying N Ritz conditions

$$\int_{\tau_1}^{\tau_2} \xi[d/d\tau,\; \tilde{y},\; x\;]\psi_k(\tau)d\tau \;=\; 0 \qquad k = 1,\ldots,N \tag{7}$$

resulting in N simultaneous algebraic equations in N unknowns.

In the present case $x(\tau)$ is periodic, so one could set

$$\tilde{y} \;=\; \sum_{1}^{3} a_k \psi_k(\tau) \;=\; Y_0 \;+\; Y_1 \cos(\nu\tau + \theta) \tag{8}$$

where Y_0 is the dc component of the video output, and Y_1 and θ are the amplitude and phase of its ripple component. A simplification is to neglect the ripple. Then all that remains is a single unknown

$$\tilde{y} \;=\; Y_0 \;=\; \Lambda V_0 \;=\; \textit{constant},$$

and only one Ritz condition is needed:

$$\int_{0}^{2\pi} \xi[\; d/d\tau,\; \tilde{y},\; x\;]d(\nu\tau) \;=\; 0. \tag{9}$$

From (2) and (3)

$$\xi[\; d/d\tau,\; \tilde{y},\; x\;] \;=\; a\left[\exp\{X\cos(\nu\tau)-[(1+b)Y_0+k\zeta]\}-1\right] \\ -\nu g\sin(\nu\tau) \;-\; a\zeta \;-\; Y_0. \tag{10}$$

Performing the integral indicated in (9):

$$\mathbb{I}_0(X) \;=\; \left(1+\zeta+\frac{Y_0}{a}\right)\; \exp\{(1+b\;)Y_0 \;+\; k\zeta\} \tag{11}$$

where $\mathbb{I}_0(X)$ is the zero-order modified Bessel function [11] of the first kind and argument X. The disappearance of the capacitance term g is a consequence of ignoring the ripple component of the video voltage. If one sets $R_L = \infty$, $I_o = 0$, $R_g = 0$, and $R_s = 0$, then this result reduces to eqn.(4) of [9].

As in linear theory the incident power is

$$P_{inc} \;=\; \frac{V_g^2}{8R_g} \tag{12}$$

so denormalizing (11) and using (12), the sought nonlinear algebraic relationship between P_{inc} and V_0 is

$$\mathbb{I}_0\left(\Lambda\sqrt{8R_g P_{inc}}\;\right) \;=\; \left(1 \;+\; \frac{I_0}{I_s} \;+\; \frac{V_0}{R_L I_s}\right)\exp\left\{\left[1 \;+\; \frac{R_g + R_s}{R_L}\right]\Lambda V_0 \;+\; \Lambda R_s I_0\right\} \tag{13}$$

This expression includes the bias current I_0 . The actual detector response can be found from (13) by calculating the *input* quantity P_{inc} as a function of the *output* quantity V_0 , an operation requiring the inverse of the modified Bessel function. In the absence of input power, the static output $V_0(0)$ is found by solving (13) for $P_{inc} = 0$, in which case $\mathbb{I}_0(0) = 1.0$. Then the *change* in V_0 in response to a finite P_{inc} is is given by

$$\Delta V_0 \;=\; V_0(P_{inc}) \;-\; V_0(0). \tag{14}$$

The family of curves in Fig. 2 was obtained from (13) and (14), using typical diode and circuit parameters. The analysis correctly predicts both "square law" and "linear" regions of operation, as well as the effect of varying the load R_L. Note that low values of R_L can lead to slopes much steeper than square-law over certain ranges of P_{inc} , an effect also seen in measured data, and in agreement with Spice simulations.

Figure 3 shows the calculated dependence of the voltage sensitivity $\gamma_v = \Delta V_0/P_{inc}$ on the incident power. The values are typical of those for real Schottky-diode detectors.

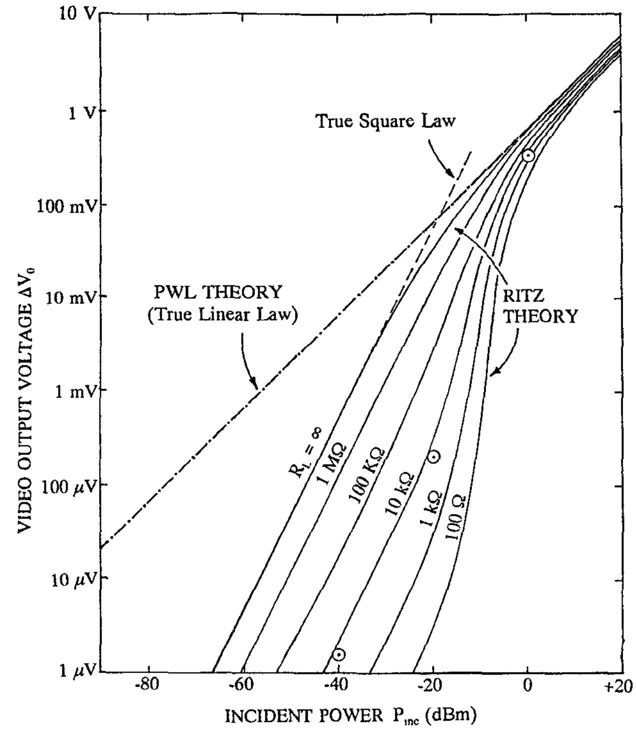

Figure 2. Transfer characteristics of a Schottky-diode video detector circuit calculated by the Ritz-Galerkin method. The effect of varying the load resistance R_L is shown. The diode parameters are

$I_s = 10^{-8} A$

$\Lambda = 40 V^{-1}$

$R_s = 10 \Omega$,

while the circuit parameters are

$R_g = 50 \Omega$

$I_0 = 0.$

The small circles indicate the results of Spice simulations for $R_L = 10 k\Omega$.

COMPARISON WITH PIECEWISE-LINEAR ANALYSIS

If the diode is represented as a resistor R_s in series with a switch that is open for reverse bias (v≤0) and closed for forward bias (i>0), then it is found that the relation between P_{inc} and V_0 is

$$V_0 = \alpha\sqrt{8R_g P_{inc}} \tag{15}$$

where $\alpha = V_0/Vg$ is the solution of

$$\sqrt{1-\alpha^2} = \alpha[\pi b + \arccos\alpha]. \tag{16}$$

The resulting true-linear-law response, towards which the responses predicted by the RG method are asymptotic for $P_{inc} \rightarrow \infty$, is also shown in Fig. 2.

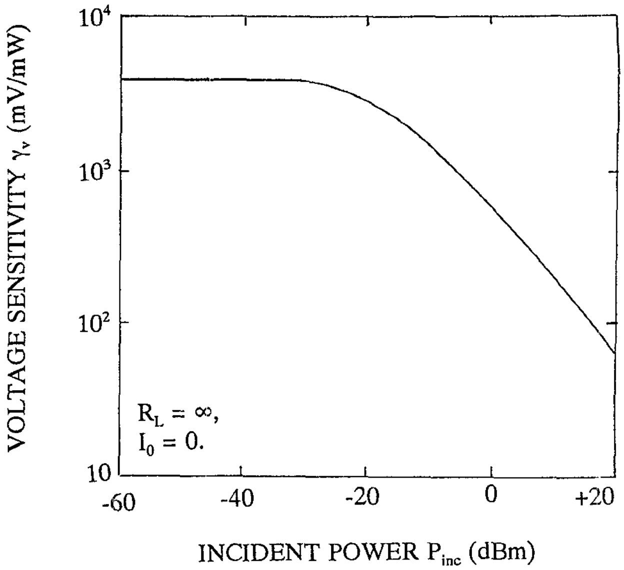

Figure 3. Calculated dependence of the voltage sensitivity γ_v on the incident power level. The device and circuit parameters are the same as for Fig. 2.

SUMMARY AND CONCLUSIONS

The Ritz-Galerkin method provides a closed-form solution for the full dynamic range of detector circuits, from "square-law" to "linear", including the effects of video loading and bias current. Greater-than-square-law slopes are seen under certain conditions, in agreement with numerical simulations. At high input levels the results approach the predictions of a piecewise-linear analysis.

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REFERENCES

[1] Ritz, W., "Über ein neue Methode zur Lösung gewisser Variations-probleme der mathematischen Physik", *J. für reine and angew. Mathematik*, vol. 135, 1908, p.1.

[2] Klotter, K., "Nonlinear Vibration Problems by Averaging Method of W. Ritz", *Proc. 1st National Congress for Applied Mathematics*, Chicago, 1951, p.125

[3] Cunningham, W.J., *Introduction to Nonlinear Analysis*, McGraw-Hill, New York, 1958, pp. 157-164.

[4] Kantorovich, L.V., and V.I. Krylov, *Approximate Methods of Higher Analysis*, P. Noordhoff, Groeningen, Netherlands, 1958, pp. 258-303.

[5] Kumagai, S., and S. Kawamoto, "Multistable Circuits using Nonlinear Reactances", *IRE Transactions on Circuit Theory*, Vol. CT-7, December 1960, pp. 432-440.

[6] Warner, F.L., "Detection of Millimeter and Submillimeter Waves", in *Microwave Semiconductor Devices and their Circuit Applications*, Iliffe, London, 1969, pp. 464-468.

[7] Wyatt, J.L., and L.O. Chua, "Nonlinear Resistive Maximum Power Theorem, with Solar Cell Application", *IEEE Trans. Circuits Syst.*, Vol. CAS-30, No. 11, November 1983, pp. 824-828.

[8] Wyatt, J.L., "Nonlinear Maximum Power Transfer Theorem", *IEEE International Symp. Circuits and Systems Digest*, May, 1983, p. 158.

[9] Wetenkamp, S., "Comparison of Single Diode versus Dual Diode Detectors for Microwave Power Detection", *IEEE MTT-S International Microwave Symposium Digest*, Boston, MA, May/June 1983, pp. 361-363.

[10] Saaty, T.L., and J. Bram, *Nonlinear Mathematics*, Dover, New York, 1981, pp. 264-265.

[11] Abramowitz, M. and I. Stegun, *Handbook of Mathematical Functions*, National Bureau of Standards, June 1964.